Selecting Minimum Factors of Safety for 3D Slope Stability Analyses

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ABSTRACT

Geotechnical engineers understand there is uncertainty and risk in the input parameters for slope stability analyses and within the analysis methodologies themselves. Decades of research and inverse analyses of slope failures have resulted in widespread acceptance of certain factors of safety (FS) in typical situations, e.g., a static two-dimensional (2D) factor of safety of 1.3 is often used for temporary or low risk slopes and 1.5 for permanent slopes. However, these FSs are not appropriate for use with three-dimensional (3D) analyses because 3D analyses account for additional shear resistance that is generated along the sides of the slide mass. The contribution of the additional shear resistance can be significant in shallow slide masses or for translational slide masses with a width to height ratio less than six, resulting in calculated values of 3D FS that are greater than the calculated 2D FS. To achieve the same level of safety or risk as a static 2D FS of 1.3 or 1.5, the user must use a greater minimum FS for 3D analyses. This paper presents methods for calculating a suitable minimum 3D FS to achieve a similar level of safety or risk as a minimum 2D FS, such as 1.3 or 1.5, would afford.

INTRODUCTION

Two-Dimensional (2D) limit equilibrium (LE) analyses are based on a plane strain condition that assumes the slide mass is infinite in both directions perpendicular to slide movement and therefore 3D effects (end effects) are negligible compared to the shear resistance mobilized across the failure surface on the bottom of the slide mass. This assumption is acceptable if the width of the slide mass is large compared to its height, i.e. ratio of width (W) to height (H) of the slide mass is greater than six (6) (Arellano and Stark 2000; Akhtar and Stark 2017). Therefore, a 2D analysis is believed to be conservative (Hutchinson and Sarma 1985; Cavounidis 1987; Hungr 1987; Duncan 1996,) for engineering purposes because the end effects are always neglected. If the W/H ratio is less than six, the ratio of 3D to 2D FS can be as high as 2.1 (Akhtar and Stark 2017). In other words, the calculated 3D FS can be twice as high as the corresponding calculated 2D FS for the same critical slide mass.

Landslides are generally not infinitely long and the geometry, pore-water pressures, and engineering properties often vary perpendicular to slide movement. Consequently, using a 2D analysis for a 3D problem is not accurate. For this reason, many engineers are using 3D stability analyses more in practice (Fredlund 2014). A 3D analysis can incorporate variations in ground surface, slope geometry, and subsurface conditions, including piezometric levels and shear strengths across the slide mass. In addition, Stark and Eid (1998) suggest a 3D analysis should always be used for inverse analyses of slope failures because 2D analyses can result in back-calculated mobilized shear strengths that are as much as 30% higher than the 3D analysis result. If the slope geometry changes significantly due to remedial measures, the inverse analysis is

important because using the 2D mobilized shear strength in a forward analysis will be unconservative (Stark and Eid 1998).

Akhtar and Stark (2017) show the 3D effects on the calculated FS are most pronounced for translational landslides because of the effects of shear resistance along vertical or near vertical sides of the slide mass parallel to the direction of slide movement. Consequently, the computed 3D factor of safety (3DFS) is underestimated which results in an overestimate of shear strength parameters through an inverse analysis. To overcome this limitation, Stark and Eid (1998), Arellano and Stark (2000), Eid et al. (2006), and Akhtar and Stark (2017) suggest different techniques to incorporate the side shear resistance in 3D LE computations because existing 3D software does not account for side shear resistance, only 3D geometry.

3D SLOPE STABILITY ANALYSES IN PRACTICE

Because of advances in computing power and speed, improvements in 3D slope stability software, e.g., 3DDEM-Slope (Akhtar and Stark 2017), engineers recognizing the limitations of 2D analyses for 3D problems, and the necessity to meet regulatory specified values of minimum FS, there is a growing interest in using 3D stability analyses. For example, design of waste containment facilities requires meeting regulatory values of FS of 1.5 for static and 1.2 to 1.3 for seismic conditions. Because disposal volume is a primary design focus, owners and designers are interested in maximizing slope inclination and slope height to maximize disposal volume. This can be accomplished, albeit at a lower level of safety, by using a 3D analysis and comparing the results to a minimum FS selected for 2D analyses, e.g., 1.5. This approach allows the designer to achieve a greater disposal volume that inherently results in a higher risk of failure. A similar situation often develops when designers are having difficulty achieving the required 2D FS values on embankments, structural fills, or cut slopes. Unfortunately, the FS values calculated using the 3D analysis are compared to acceptable minimum FS values selected for comparison to FSs calculated using 2D analyses. In either case, comparing the FS calculated using a 3D analysis to a specified minimum 2D FS value results in greater uncertainty and risk.

Over decades of practice and inverse analyses of observed failures, the profession has adopted minimum values of FS suitable to limit slope failures. These adopted minimum FSs were developed for comparison to static FSs generated by 2D analyses in various situations, e.g., 1.3 is often specified for temporary or low risk slopes and 1.5 for permanent slopes. For example, the U.S. Army Corps of Engineers (USACE) Engineering Manual EM 1110-2-2502 *Retaining and Flood Walls* (1989) provides minimum factors of safety of 1.5 and 1.1 for coastal flood walls under static and seismic loading conditions, respectively. These accepted minimum FSs were developed using 2D calculation methods and should only be compared to FSs calculated using a 2D method not a 3D method. This paper presents a method for estimating a new minimum static 3D FS that corresponds to the same level of safety as a regulatory minimum 2D FS, e.g., 1.5.

UNCERTAINTY IN SLOPE STABILITY CALCULATIONS

Regardless of the level of sophistication, slope stability analyses simplify complex field conditions into manageable simulations to provide an estimate of slope stability. For this reason, an understanding and quantification of the uncertainty in a stability analysis is important because of the significant variability in geotechnical material (predominantly soil) properties and pore-water pressures. Uncertainty in slope stability analyses stems from a variety of factors, including:

1. Inherent variability of input parameters,
2. Difficulty in measuring input parameters,
3. Improperly selected critical failure surface, and
4. Limitations of stability analyses to represent complex subsurface conditions.

Even if the parameter selection is appropriate and the problem is modeled well, slope movement has been observed to initiate at a FS of up to 1.05 (Hussain and Stark 2010). This would indicate that limitations in stability analyses contribute at least 5% of the overall uncertainty in a slope stability calculation, which leaves the remaining 95% for the other three factors listed above. The engineer has little control over Factor #1 above, but should include additional FS to cover known variability in material properties (Factor #2). Engineering judgment and experience are needed to ensure that the uncertainty in Factors #3 and 4 is minimized to the extent practical.

The uncertainty created by the four factors above can be quantified using the reliability analysis developed by Duncan (2000). This method captures the effect of soil variability and differences in test procedures and the resulting probabilities of failure can be estimated using an approximate technique based on the Taylor Series method. In this method, the coefficient of variation, i.e. the ratio of the standard deviation to the mean, of key soil properties, e.g., shear strength and unit weight, are used to estimate the probability of failure associated with soil property variation, i.e., the level of uncertainty in soil parameters. In fact, Duncan (2000) uses a slope stability case history to illustrate this method, which is discussed below.

The first step in this reliability analysis is to estimate the standard deviation in the parameters impacting the computed 2D or 3D FS. A Taylor Series is used to estimate the standard deviation and variance in FS based on the change in FS caused by the standard deviation (SD) in all of the parameters that influence FS (Duncan 2000). The standard deviation in the FS (σ_F) is estimated using the following Taylor Series expression:

$$\sigma_F = \sqrt{\left(\frac{\Delta F_1}{2}\right)^2 + \left(\frac{\Delta F_2}{2}\right)^2 + \left(\frac{\Delta F_3}{2}\right)^2} \tag{1}$$

where, ΔF is the change in FS computed for the most likely value (MLV) plus one SD (+1SD) and the MLV minus one SD (-1SD) for the parameter in question, e.g., soil unit weight. Thus, the change in FS for each parameter that corresponds to the MLV +1SD and MLV -1SD is estimated and used to compute σ_F using Equation (1).

The Three-Sigma Rule is generally used to characterize the variability of soil properties. The Three-Sigma Rule is based on the fact that 99.7% of all values of a normally distributed parameter fall within three standard deviations of the mean (Duncan 2000). Therefore, the Three-Sigma Rule can be used to estimate a value of standard deviation by first estimating the highest and lowest conceivable values of a parameter and then dividing the difference between them by six (Duncan 2000) as shown below:

$$\sigma = \frac{HCV - LCV}{6} \tag{2}$$

where, HCV is highest conceivable value of the parameter and LCV is the lowest conceivable value of the parameter.

RELATIONSHIP BETWEEN 3D AND 2D FS

Figure 1 presents a relationship between the ratio of 3D/2D FS ($R_{3D/2D}$) and W/H for three slope inclinations considered in the parametric study conducted by Akhtar and Stark (2017). These relationships are for effective stress friction angles of 30° and 8° for the upper and lower materials, respectively, and supersede the relationships presented in Arellano and Stark (2000). A review of Figure 1 shows that for 1H:1V slopes, the $R_{3D/2D}$ is less than about 1.3 for all W/H ratios, even those less than 6. Conversely, the difference in $R_{3D/2D}$ is much greater, up to 2.1, for flatter slopes, e.g. a 5H:1V slope inclination. This larger ratio is caused by the greater area along the sides of the shear mass for flatter than steeper slopes which results in a greater amount of side shear resistance. Further, the maximum value of $R_{3D/2D}$ (~2.1) occurs at a W/H ratio of one (unity). Expectedly, the $R_{3D/2D}$ for a 3H:1V slope fall between those for the 5H:1V and 1H:1V slopes. For more information about the translational slope model used to develop Figure 1, the reader is referred to Akhtar and Stark (2017).
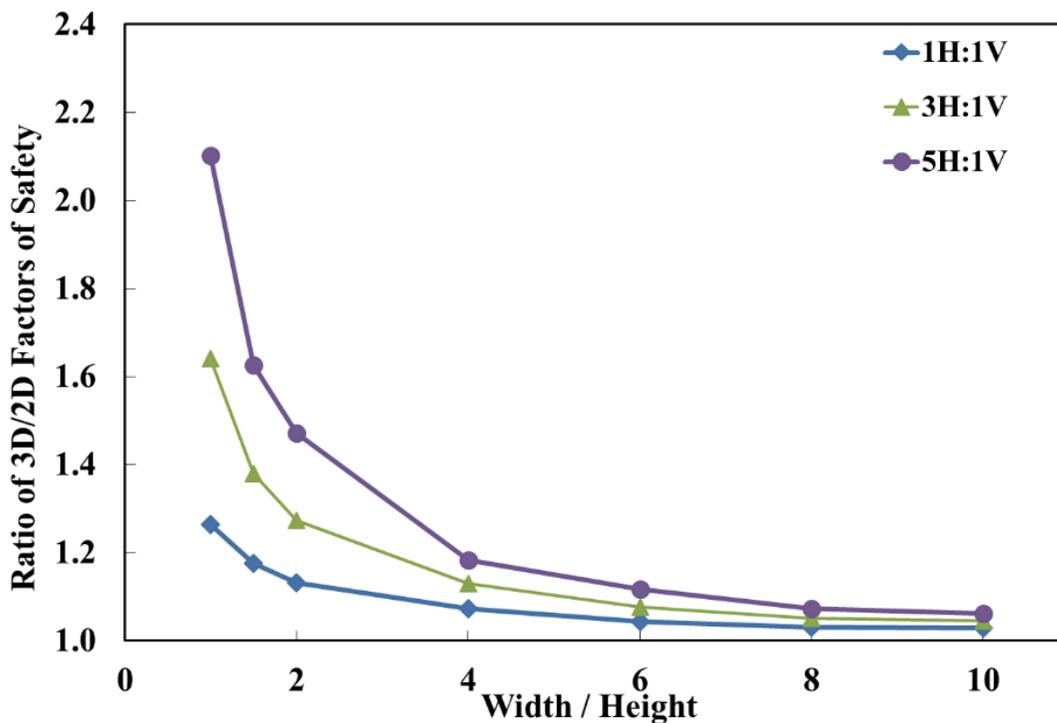

Figure 1. **Relationship between 3D and 2D FS ($R_{3D/2D}$) as a Function of Slide Configurations (from Akhtar and Stark 2017)**

RECOMMENDED CALCULATIONS OF MINIMUM FS FOR 3D ANALYSES

Figure 1, or a similar plot developed for a specific slope, can be used to estimate $R_{3D/2D}$ for a given W/H ratio and slope inclination. The resulting ratio can be used to estimate the 3D FS that will

yield the same level of safety as a 2D FS, e.g., 1.5. Three approaches are presented below for the calculation of the minimum 3D FS with three different levels of safety.

In the most conservative case, the $R_{3D/2D}$ from Figure 1 (or a similar site specific plot) can be multiplied by the minimum required 2D FS, e.g., 1.5, to determine the minimum FS for 3D analyses (FS_{3Dm}), as shown in Eq. 3:

$$FS_{3Dm} = FS_{2Dm} \times R_{3D/2D} \tag{3}$$

where, FS_{2Dm} is the minimum allowable FS for comparison with the result of a 2D analysis, e.g., 1.5, and FS_{3Dm} is the minimum allowable FS for comparison to the result of a 3D analysis. Eq. (3) negates the additional sophistication of a 3D analysis method by simply multiplying the minimum required 2D FS by $R_{3D/2D}$ from Figure 1, which may be appropriate for use in high risk situations or where there is large uncertainty in the soil parameters.

Alternatively, if the generally accepted minimum FSs for 2D slope stability calculations are representative of the overall risk tolerance for slope stability calculations, i.e., inclusive of soil parameter variability, then these values, coupled with an understanding of the additional uncertainty imparted by a 3D analysis (represented by an approximate ratio of the additional side shear area to the bottom shear area), can be used to calculate an alternate minimum 3D FS suitable for direct comparison to the results of 3D analyses. This approach uses the minimum 2D FS to represent inherent uncertainty and $R_{3D/2D}$, e.g., Figure 1, to account for the additional uncertainty imparted by a 3D analysis. As a result, FS_{3Dm} can be computed using Eq. (4):

$$FS_{3Dm} = FS_{2Dm} + \frac{FS_{3D} - FS_{2Dm}}{FS_{2Dm}} \tag{4}$$

A simplified version of Eq. (4), including $R_{3D/2D}$, is shown in Eq. (5):

$$FS_{3Dm} = FS_{2Dm} + R_{3D/2D} - 1 \tag{5}$$

Finally, a slightly less conservative minimum FS for comparison to the results of FSs determined using 3D analyses can be computed using an alternate version of Eq. (4) whereby the denominator is replaced by the 3D FS, resulting in Eq. (6):

$$FS_{3Dm} = FS_{2Dm} + \frac{FS_{3D} - FS_{2Dm}}{FS_{3D}} \tag{6}$$

Assuming the 2D FS in $R_{3D/2D}$ is the minimum 2D FS, a simplified version of Equation (6) is shown in Eq. (7):

$$FS_{3Dm} = \frac{R_{3D/2D} - 1}{R_{3D/2D}} + FS_{2Dm} \tag{7}$$

Equations (3), (5), and (7) can be used to calculate a recommended minimum 3D FS for comparison to the results of 3D analyses, but all three equations require an understanding of $R_{3D/2D}$ for the slope in question as well as the target minimum 2D FS. The methods in Eqs. (5) and (7) allow the user to account for some of the additional sophistication afforded by a 3D analysis without decreasing the overall level of safety.

PRACTICAL CONSIDERATIONS FOR EQUATION SELECTION

Every design situation requires a unique approach that considers the inherent risk associated with soil property uncertainty, problem complexity, and failure consequences. As outlined by the US EPA (EPA 1988), different minimum FSs are appropriate for different design and risk situations. Following that logic, Table 1 presents recommended minimum FSs for design scenarios that Equations (3), (5), and (7) could be used to determine the minimum 3D FS value for comparison to the 3D FS produced by a 3D analyses. The minimum 3D FSs presented in Table 1 were calculated using the Eqs. (3), (5), and (7) and are shown in the last column (in parentheses) using the $R_{3D/2D}$ from Figure 1 for a hypothetical 3H/1V slope with a W/H of 4.

Table 1. Recommended 3D FS for Various Scenarios[1]

Soil Strength Uncertainty	Imminent Threat to Human Life	Potential for Major Construction or Environmental Impact	Recommended minimum 2D FS[2]	Minimum 3D FS and (Equation)[3]
small	low	low	1.3	1.4 (7)
small	high	high	1.5	1.7 (5)
small	low	high	1.3	1.5 (5)
small	high	low	1.5	1.7 (3)
large	low	low	1.5	1.7 (5)
large	high	high	2.0	2.3 (3)
large	low	high	1.5	1.7 (3)
large	high	low	2.0	2.3 (3)

Table 1 Notes:
[1] All generic scenarios are representative of long term or permanent static applications where Figure 1 is representative of the ratio of the 3D to 2D FS. It may be appropriate to use alternate minimum FSs in short term or temporary applications.
[2] Only use the recommended values in Table 1 in the absence of site specific values or applicable regulatory minimums.
3 All values have been rounded to present an FS with two significant figures. Equation 3 will always produce the most conservative 3D FS, followed by Eq. (5), then Eq. (7). Always use engineering judgment when selecting which equation to select.

MINIMUM 3D FS EXAMPLE

As an example, Duncan (2000) describes the construction of a new Lighter Aboard Ship (LASH) terminal at the Port of San Francisco in 1970. Part of the LASH terminal construction included underwater excavation for a 30 m (100 ft) tall stability berm. Post excavation, the underwater excavated area was to be filled with sand to serve as the stability berm. The calculated 2D FS for the underwater excavation with side slopes of 1H:1V was 1.25 which was consistent with observations of many excavations in the San Francisco Bay mud that were safely completed with side slopes of 1H:1V. In evaluating the planned excavation side slopes, the designer determined that side slopes of 0.875H:1V could be safe and save the project approximately $200,000 (approximately $1.2M in 2016). The calculated 2D FS for the steeper side slopes (0.875H:1V) was 1.17. In consideration of the large amount of high quality data used in the FS calculation, the steeper side slopes were considered acceptable and selected for use. On August

20, 1970, a 75 m (250 ft) long portion of the underwater slope failed during excavation (Duncan and Buchignani 1973). The failure occurred within the San Francisco Bay mud with an undrained shear strength of 5 to 50,000 kPa (0.1 to 1,000 ksf). The remainder of the 600 m (2,000 ft) long excavation remained stable until it was backfilled with sand to create the stability berm nearly 4 months later. The post failure investigation determined that the failure was caused by a decrease in the Bay mud shear strength associated with creep strength loss. This creep strength loss was not observed in the laboratory or field shear strength tests which were all performed at relatively rapid shearing rates, when compared to those observed in the field.

If a 3D stability analysis was performed for this design were performed to increase the FS for the 0.875H:1V slope from 1.17, the 3D FS would need to be compared to a value greater than 1.25. First, Table 1 can be used to estimate the minimum 3D FS that should be used instead of 1.25. At the time of the design and expected construction, the underwater slope had little imminent threat to human life, a potential major construction or environmental impact, and there was thought to be small uncertainty in the shear strength parameters. As a result, Table 1 indicates that a FS of 1.3 would be appropriate for a 2D analysis and that Equation (5) can be used to estimate the minimum 3D FS. Using Figure 1, and assuming a hypothetical 1H:1V slope with a W/H of 2 to simulate the underwater excavated slope, the $R_{3D/2D}$ is about 1.15. Using Equation (5), the minimum 3D FS should be 1.45 for comparison with 3D analyses. In hindsight, knowing that a portion of the slope failed due to material property variability, it may have been more appropriate to assume a large uncertainty in the shear strength parameters, e.g. to account for the effects of soil creep. In this case, Table 1 indicates that Equation (3) would be appropriate for calculating the minimum 3D FS and that the minimum 2D FS should be 1.5. Using Figure 1 and assuming the same hypothetical slope, Equation (3) indicates that the minimum 3D FS would be 1.73. Note, in practice, the user would use Duncan's method to determine the σ_F specifically for the 3D analyses to possibly select a higher FS consistent with a probability of failure similar to a 2D analysis.

SUMMARY AND RECOMMENDATIONS

The following summary and recommendations are presented based on the comparison of 3D and 2D slope stability analyses presented above:

- The ratio of a FS calculated using a 3D analysis method to a FS calculated using a 2D analysis method for the same slope will always be greater than unity (1) for the critical failure surface.
- Flatter slopes have higher $R_{3D/2D}$ values because of the larger side area and side resistance than steeper slopes with the same slope height.
- Factors of safety calculated using 3D methods cannot be directly compared to 2D regulatory minimum FS values, e.g., 1.5, because this will result in a lower level of safety due to the additional side resistance. A greater minimum FS should be used to assess 3D analyses to create the same level of safety as a 2D FS such as 1.5.
- Three methodologies are presented herein for estimating a minimum 3D FS that corresponds to a similar level of safety as widely used 2D FSs, e.g., 1.5. The most conservative approach, Eq. (3), negates the additional sophistication of a 3D analysis method, but may be appropriate for use in high risk situations or when soil parameters are not well understood. The other methods, Eqs. (5) and (7) allow the user to account for some of the additional sophistication afforded by a 3D analysis without increasing the overall level of safety.

- The uncertainty in a 3D analysis created by variability in the input parameters can be quantified using the 2D reliability procedure presented by Duncan (2000).
- 3D slope stability modelling is still rather complicated and most suited for inverse analyses of slope failures. As more 3D stability software packages are developed that can be appropriately used to perform 3D analyses, the use of 3D analyses will expand. The complexities and variables affecting 3D analysis accuracy, e.g. mesh size, are not addressed in this paper.

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